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Javier Legris

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On The Epistemological Justification of Hilbert's Metamathematics

Javier Legris
University of Buenos Aires and CONICET (Argentina).

Abstract: The aim of this paper is to examine the idea of metamathematical deduction in Hilbert's program showing its dependence of epistemological notions, specially the notion of intuitive knowledge. It will be argued that two levels of foundations of deduction can be found in the last stages (in the 1920s) of Hilbert's Program. The first level is related to the reduction – in a particular sense – of mathematics to formal systems, which are 'metamathematically' justified in terms of symbolic manipulation. The second level of foundation consists in warranting epistemologically the validity of the combinatory processes underlying the symbolic manipulation in metamathematics. In this level the justification was carried out with the aid of notions from modern epistemology, particularly the notion of intuition. Finally, some problems concerning Hilbert's use of this notion will be shown and it will be compared with Brouwer's notion and with the idea of symbolic construction due to Hermann Weyl.

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This paper deals with the interplay between logic and philosophy in Hilbert's metamathematical program. It examines the idea of metamathematical deduction in Hilbert's program showing its dependence of epistemological notions, specially the notion of intuitive knowledge. The distinction will be made between *two foundational levels* or layers which can in fact be observed in the last stages (in the 1920s) of Hilbert's Program. The *first* level is related to the reduction in a particular sense of mathematics to formal systems, which are 'metamathematically' justified in terms of symbolic manipulation. This metamathematical deduction led to a characterization of logic as a theory of formal deductions that was fundamental for the historical development of the proof-theoretical understanding of logic. The *second* level of foundation consists in warranting epistemologically the validity of the combinatory processes underlying the symbolic manipulation in metamathematics. In this level the justification was carried out with the aid of notions from modern epistemology, particularly the notion of intuition. Finally, some problems concerning Hilbert's use of this notion will be shown and it will be compared with Brouwer's notion and with the idea of symbolic construction due to Herrmann Weyl.

I

As generally known, Hilbert's foundational program sets as its goal the achievement of consistency proofs for mathematical theories. This goal was pursued in every stage of it, from *The Foundations of Geometry* of 1899 to the *Foundations of Mathematics* (Die *Grundlagen der Mathematik*) of 1934 and 1938.In a certain sense, consistency seemed to warrant the existence of mathematical objects and structures. This supposition required notwithstanding some epistemological justification. Now, Hilbert began what Vittorio Michele Abrusci called the *fourth stage* (at the beginning of the 1920s) of his program with a criticism of Brouwer's Intuitionism and Weyl's related conception on the foundations of mathematics (see [Abrusci 1978, 31 ff.]). He critized especially Brouwer's idea of grounding mathematics on the Primordial Intuition of Time in which the sequence of natural numbers is given (see [Hilbert 1922]). According to Hilbert such a subjective capacity could not be the basis for mathematical knowledge. His own proposal to solve the *Grundlagenkrise* in mathematics relied on *symbolic manipulation* as an intersubjective and unquestionable basis for reconstructing mathematical theories by means of consistency proofs. This approach presupposed the epistemological moment of formal abstraction from every content. This symbolic manipulation constituted *metamathematics*, and its warrant laid in the finite point of view (or finitism). This conception established which notions and statements were acceptable within the bounds of as Hilbert and Bernays stated it in their opus magnum *Die Gundlagen der Mathematik* the conceivability in principle of objects and the execution in principle of procedures [Hilbert & Bernays 1934, 32].

Thus, formal axiomatic systems were constituted by symbols considered as merely *physical* objects, and proof procedures were procedures for symbolic manipulation. At a first sight, the idea was very simple, reasonable and it was consistent with common sense. As Hilbert stressed,

> if mathematics is to be rigorous, only a finite number of inferences is admissible in a proof - as if anyone had ever succeeded in carrying out an infinite number of them [Hilbert 1926, 370].

Obviously, logical rules should also be counted as rules for procedures of this kind. As Bernays expressed it in a paper written for a philosophical audience (I am referring to the paper "Die Philosophie der Mathematik und die Hilbertsche Beweistheorie", from 1930), a proof should be essentially characterized by the moment of *combination*. The *logical* nature of inference rules, which it is shown by the *real execution* of the proof, relies on this moment (see [Bernays 1930a, 334]). Logic is then interpreted as the theory of formal deductions, an idea that would be developed later in the framework of proof- theoretical accounts of logic. These combination procedures needed an epistemological justification.

This idea of founding mathematics on symbolic manipulation was discussed within Hilbert's circle of collaborators. For the question arose how these procedures can be justified. It could be argued that the development of a method for proving consistency of axiomatic systems was properly a methodological goal. However, Hilbert himself (and also Bernays) aimed to a philosophical framework for his program. Indeed, it was noticed that the very notion of symbolic manipulation required a philosophical justification of its own, that is, to answer the question: What led Hilbert to take formal (symbolic) systems as a source of a secure and indubitable knowledge, since the *abstraktes Operieren* with concepts and contents showed itself as uncertain and unreliable? In other words, the epistemological reliability problem of the finitist method itself was posed in Hilbert's circle.

A possible solution to this problem could start from the notions of symbolic manipulation and consistency themselves, arriving at some kind of ‘intrasymbolic’ justification. It could be regarded as a *pragmatic* justification of some kind. The rules generating the formal system should be viewed as defining an object or a structure. A solution of this kind could be rooted in some sense in a philosophical tradition stemming from Leibniz and it was, in fact, suggested by Hilbert in the first stage of his program before the metamathematical turn. This point of view will be also be found in Weyl’s foundational work (as it will be shown later).

However, Hilbert (together with Bernays at that time) explored another solution based on the capacities of the knowing subject. This decision was influenced by the philosophical context in which they were situated, mainly in the kantian tradition, and perhaps by the discussion with Brouwer. Finally, the desired justification was found in the notion of intuition [*Anschauung*]. In his conference on the new foundation of mathematics Hilbert stated:

> Instead, as a precondition for the application of logical inferences and for the activation of logical operations, something must already be given in representation [*in der Vorstelllung*]: certain extra-logical discrete objects, which exists intuitively [*anschaulich*] as immediate experience [*Erlebnis*] before all thought. If logical inference is to be certain, then these objects must be capable of being completely surveyed in all their parts, and their presentation, their difference, their succession (like the objects themselves) must exist for us immediately, intuitively, as something that cannot be reduced to something else. [Hilbert 1922, 162 f.](English translation in [Mancosu 1998, 202].)

This statement will be repeated in future writings. His famous paper “On The Infinite”, reads as follows:

> ... as a condition for the use of logical inferences and the perfomance of logical operations, something must already be given to our faculty of representation, certain extralogical concrete objects that are intuitively present as immediate experience prior to all thought. [Hilbert 1926, 376] (see also [Hilbert 1931, 486]).

The “concrete objects” intuited were symbols and every manipulation of symbols should also be intuited. In this sense, logic inference itself must also be based on intuition as well, as Bernays observed in a paper also aimed to a broad philosophical audience (“Die Grundgedanken der Friest́schen Philosophie in ihrem Verhältnis zum heutigen Stand der Wissenschaft” from 1930):

> ..., daß man schon die Logik als Theorie der Urteile uns Schlüsse gar nicht ohne eine gewisse Heranziehung einer solchen anschaulichen Erkenntnis entwickeln kann. Es handelt sich dabei um die anschauliche Vorstellung des Diskreten, aus der wir die primitivsten kombinatorischen Vorstellungen, insbesondere die der Sukzession entnehmen. [Bernays 1930b, 108]

The formal proofs of metamathematics are also intuitively grasped. (As Hilbert stated in a later paper: "Ein Beweis ist eine Figur, die uns als anschaulich vorliegen muß" [Hilbert 1931, 489].)

It is generally acknowledged that intuition plays an essential role in the constitution of Hilbert's finitism. Kurt Gödel in his *Dialectica* paper of 1958 defined finitary mathematics as that of intuitive evidence. Hourya Sinaceur made use of the epistemological relation between intuition and finitism in order to distinguish it clearly from the axiomatic method: The finite logic of Hilbert must proceed intuitively [Sinaceur 1993, 252]. Charles Parsons also related both notions, stressing that the main epistemological thesis of Hilbert's program should rely on intuition (see [Parsons 1998]). Now, the main point is to understand accurately (i) what Hilbert really meant by intuitions and (ii) why he chose this notion for the philosophical grounding of his finitism.

II

The notion of intuition has received diverse interpretations in the history of philosophy. Intuition was always taken as a faculty or capacity of knowledge which is applied, in the first place, to objects: Intuition of objects is a direct grasping of them as in perception. This grasping can be sensible or intellectual. In a second sense, intuition serves to determine the *truth* of statements or propositions. This is the case of self-evident truths: the evidence for the truth of a sentence is given by means of intuition. So, intuition comes to be a method for *justifying* the truth of sentences. Obviously, if the truth of a sentence is justified, then its truth can be asserted. For this reason, intuition is regarded as a capacity of knowledge determining the *truth* of sentences.

The second sense relies on the first: sentences whose truth is justified by intuition refer to objects intuitively grasped. These intuitions should be necessary conditions for the validity of logical deductions. These intuition-grasped objects constituted the reliable and secure core for finitary arithmetics, and there is no doubt that, according to Hilbert, they were tokens, concrete symbols without meaning, such as strokes or symbols like the string 1+1+1+1 deprived of its standard meaning (see [Hilbert 1922, 163] and also [Hilbert 1926]). The combination of these symbols by means of operations is the basis of finitistic arithmetic. From this approach, an epistemological thesis underlying Hilbert's program can be rendered in the following two statements:

(1) If A is a true sentence of finitary arithmetic, then its truth can be inferred from sentences whose truth is justified by intuition.

(2) Finitary arithmetic is intuitive arithmetic.

Both of them can be accepted as the main epistemological thesis underlying Hilbert's finitism.

At this point it must be clarified in what sense deductions made according the finite point of view are *intuitive*. With regard to this application of the notion of intuition, two different senses are traditionally taken into account: (i) as applied to the justification of the truth of sentences, and (ii) as applied to the justification of the correction for deduction processes. In the first case, deduction provides an intuitive justification of true *sentences*, that is, the last sentence of a deduction is also justified by induction. In the second case, it is the *process* of rule application constituting the deduction which is justified by intuition. The first sense is tenable only if one thinks that deduction preserve the intuitive character of sentences: If the premises of a deduction are intuitively known, so is the conclusion.

This sense is highly debatable because of the fact that deduction is not an *immediate* way of recognizing the truth of a sentence. It can be observed that, classically, this sense was already discussed by Descartes in Rule II of his *Regulae ad directionem ingenii*. Descartes distinguished between intuition and deduction by indicating in the latter the existence of a movement or succession of thinking. However, this movement intuits each thing in particular (see [Descartes 1701, AT, X, 369]) and the chain of steps brings the certainty of the last point. Each element grasped in this movement is intuited. This leads to the second sense. The correctness of each step in the deduction is justified by intuition. And so, the intuitive character of the whole process can only be asserted in a secondary or derivated sense. To sum up, it is not the final result of the deduction that is known by intuition, but it is each step that is intuitively justified. This notion of intuition is used to characterized the validity or correctness of each step in a deduction: the validity not only of the basic rules but also of their application are justified by intuition.

Obviously, even if Hilbert's idea of intuition fits in the second sense, some obscurities remain. For the notion of intuition did not receive in Hilbert's papers a proper analysis or elucidation. Of course, its relation to finitary arithmetic gives us some clue: the potential infinite, for example, cannot be grasped by intuition, and in this point lies an essential difference with Brouwer's intuitionism.

Moreover, due to the historical background, the philosophical discussions in Hilbert's circle made use of notions from the Kantian tradition, particularly "the pure forms of sensibility": the pure intuitions of space and time. However, intuition can be better understood as *empirical perception*, as symbols are concrete objects to be grasped by our senses.(Kantian influence should be found better in Hilbert's conception of *ideal mathematics*.)

III

The deductive procedures in metamathematics also receive a justification by intuition. In a few words, in Hilbert's metamathematics, logical symbols represent operations performed on symbols themselves, so that logical deduction is based on intuitive combinatorial principles. Thus, it is possible to talk about a logic of finitary processes, a "concrete" logic which is the subject of metamathematics. This leads to the idea of a *metamathematical meaning* of logical symbols diverging from the ordinary meaning: It is the meaning they have in formal proofs constructed according to the finite point of view. In a paper of 1927, aimed to a broad audience, Bernays regarded as universally valid rules such as the *dictum de omni et nullo*, the modus ponens, the laws of non contradiction and excluded middle and the principle of proof by cases. This acceptance of the excluded middle implied the unproblematic character of finitary negation. Hilbert himself expressed the same ideas (see [Hilbert 1923, 181]).

However, as it is well known, Hilbert himself saw quantified formulas as representing the knowledge about infinite aggregates, departing therefore from the realm of intuitive and finite thinking. This conviction was motivated by Hermann Weyl's rejection of the principle of excluded middle due to his analysis of the meaning of quantified formulas in his paper on the new foundational crisis of mathematics (see [Weyl 1921]) . So, Hilbert wrote

> Wo geschieht nun zum erstenmal das Hinausgehen über das konkret Anschauliche und Finite? Offenbar schon bei der Anwendung der Begriffe 'alle' und 'es gibt'. [Hilbert 1923, 181].

Although in Hilbert's papers from that time we can find some confusion between metamathematical quantification and quantification over finite domains (see loc. cit.), Hilbert explored from 1923 onwards different ways to define metamathematical quantifiers. Later, in the *Grundlagen der Mathematik*, an universal judgement was interpreted finitarily as an hypothetical judgement, whereas an existential judgement as a "partial judgement" (*Partialurteil*), that is as an incomplete judgement. So, they could not be considered as authentic judgements. Now, in the axiomatic system he introduced in his paper of 1923 firstly occur quantifier-free number theoretic axioms, and then in a separate way what he used to call transfinite axioms introducing quantification. It must be noticed that the finite point of view led to the exclusion of notions like arbitrary interpretation and satisfiability, which serve usually to interpret quantification.

Basically, he introduced a choice function, the τ function, which applied to predicates served to choose an arbitrary element of the domain. By means of this function, universal quantifier could be defined. In fact, Hilbert called it a transfinite function (see [Hilbert 1923, 183]). Later, in his paper "On the infinite", he proceeded in another fashion introducing a choice function ϵ, and $\exists x Ax$ denotes an indeterminate function that pick out an x for which A holds. In other words, this expression means an x such that if anything has the property A, then x has that property. The existential quantifier could be then defined explicitly as

$$\exists x Ax =_{df} A(\epsilon x A)$$

Now, the terms obtained by both functions can only be applied a finite number of times in a formal deduction, and consequently, the function could be finitarily interpreted (see the comments in [Goldfarb 1979, 361] and a full systematical account is to be found in [Leisenring 1969]).

Such a conception of quantification should be enough for Hilbert's purposes. The choice functions expressed the meaning quantifiers had within metamathematics. But, it cannot be sustained that these functions convey necessarily an intuitive idea of quantification. Intuition serves only to justify metamathematical procedures, and therefore the use of logical symbols in *formal proofs*, but it does not elucidate the meaning of logical symbols in general. This idea can be accurately illustrated through a passage extracted from Bernay's paper "Die Philosophie der Mathematik und die Hilbertsche Beweistheorie". This passage is about the application of modus ponens in a formal derivation [Bernays 1930a, 335]. Let A and A * B be two formulas of the formal language, which are not axioms. Then, there exists a sequence S of formulas leading to A and another sequence T leading to A * B, and the rule of modus ponens allows the derivation of B. Thus, the purely formal coincidence between the last formula of the sequence T and the formula obtained from the last formula of S as antecedent and the derived formula as consequent:

> The coincidence, which is to be found in the present case, cannot be directly read off from the content of the formal inferences rules and from the structure of the initial formulas, but rather it can be only be read off from that structure obtained through the application of the inference rules, that is, through the carrying out of the inferences. There exists here in fact a combinatorial element. (*loc. cit.*, (English translation in [Mancosu 1998, 241])

IV

It can be observed that *two levels* coexist in the foundational arguments of Hilbert's Program. The *first* level includes the reduction of mathematics to formal systems, which are justified 'metamathematically', in terms of symbolic manipulation. This reduction is carried out in a very special way, which has always been controversial and will not be discussed here. It is parallel to the reduction of ideal mathematics into finitary mathematics. It was in this level of foundation that Hilbert found a solid basis for mathematical knowledge, attempting to solve the foundational crisis. The *second* level of foundation consists in warranting the validity of the combinatory processes underlying the symbolic manipulation in metamathematics. This level is more basic, connected with perception of symbols and the operations carried out with them. It is at this level where the notion of intuition appears.

The differentiation between these two levels can be suggested in the discussion that took place after the lecture "Kritische Philosophie und mathematische Axiomatik" given by the philosopher Leonard Nelson in 1927 in Göttingen. For Nelson, the function of consistency proofs was basically systematic and methodological, that is, the critical representation of a logical relation between the axioms of a mathematical theory. On the other side, the axioms should be justified by pure intuition ("gerechtfertigt werden die Axiome durch Berufung auf die reine Anschauung", [Nelson 1959, 122]). Bernays agreed with Nelson on the role of intuition in mathematics, but he strongly objected the idea of founding mathematics axioms on the kantian pure intuitions of space and time [Nelson 1959, 110]. Nelson replied that metamathematics must rest on an a priori intuition, namely the spatial intuition, since symbols are extensive figures, given in a spatial order. Otherwise, if some kind of empirical intuition were the basis, like the perception of the marks made by a chalk on the blackboard, certain apodictic principles about symbols should be needed. This is what Nelson ironically called the metaphysics of chalk [Nelson 1959, 118]. This discussion shows the difficulties caused by the introduction of intuition and reveals to what extent the notion of intuition was not explicitly analyzed in Hilbert's epistemological arguments.

Kant's transcendental idealism established that intuition should be some kind of construction in the subjectt's mind. As it is known, L. E. J. Brouwer based his mathematical intuitionism on the idea of intuition as construction. The 'primordial intuition' of the 'two-oneness' in time, the repetition of objects in time is what generates the sequence of finite ordinal numbers as mental constructions of the mathematical subject. The *locus classicus* is Brouwer's doctoral dissertation [Brouwer 1907].

Obviously, Hilbert's and Brouwer's ideas of intuition are quite apart one from another. Hilbert made use of intuition in order to justify only his metamathematics, while Brouwer pursued the idea of founding the entire building of mathematics through intuition, leading him to the well known restrictions in mathematical methods. Nevertheless, in another aspects they are quite close. In fact, the idea of construction is essential to Hilbert's metamathematics, but it is a *symbolic construction*: deductions and proofs are thought in terms of symbolic operations or constructions. The justification of metamathematics relies here on the notion of *symbolic construction* itself and, more generally, on the human capacity of constructing symbolic systems. Besides, it can be said that these two levels mentioned above could have been unified in this idea of symbolic construction.

This approach was sketched by Hermann Weyl in a philosophical paper from the 1920s [Weyl 1925]. The main idea can be summarized as follows. Weyl aimed to provide with a sense for the whole system of mathematics ("mathematics as totality"), including transfinite mathematics, which is impossible to understand intuitively and which was also called by Weyl "theoretical mathematics". According to him, this "transcendent" part of mathematics could only represented by means of symbols. Therefore, its knowledge would be some kind of "symbolic knowledge" (even if this expression does not occur explicitly in Weyl's papers). And it could be regarded as a *pragmatic* point of view. He wrote:

> Without doubt, if mathematics is to remain a serious cultural concern, then some sense must be attached to Hilbert's game of formulae, and I see only *one* possibility of attributing it (including its transfinite components) an independent intellectual meaning.[...] Theories permit consciousness to jump over its own shadow, to leave the matter of the given, to represent the transcendent, yet, as is self-evident [*wie sich von selbst versteht*], only in *symbols*. [Weyl 1925, 540] (English translation in [Mancosu 1998, 140])

> If Hilbert is not just playing a game of formulae, then he aspires to a theoretical mathematics in contrast to Brouwer's intuitive one. [...] Yet beside Brouwer's way, one will also have to pursue that for Hilbert; it is undeniable that there is a theoretical need, simply incomprehensible from the merely phenomenal point of view, with a creative urge directed upon the symbolic representation of the transcendent, which demands to be satisfied. [Weyl 1925, 541] (English translation in [Mancosu 1998, 140])

He argued for similar ideas in philosophical papers published later, in the late forties and the fifties: In the construction procedures, the idea of (potential) infinity is already included:

> Die Zeichen werden nicht einzeln für das jeweils aktuell Gegebene hergestellt, sondern sie werden dem potentiellen Vorrat einer nach festem Verfahren herstellbaren, geordeneten, ins Unendlichen offnen Mannigfaltigkeit vor Zeichen entnommen. [Weyl 1953, 223]

It is the application of the symbolic system to natural science that makes it reliable:

> Wenn die formale Mathematik nicht mehr den Anspruch erhebt, wahre Behauptungen aufzustellen, so muß man sich fragen, was sie dann überhaupt bezweckt [...] überzeugender ist der Hinweis auf ihren naturwissenschaftlichen Gebrauch, auf die Rolle, die sie beim konstruktiven Aufbau einer Theorie der wirklichen Welt durch die Physik spielt. Denn hier

können wir uns auf die Bewährung der theoretischen Konstruktion durch Erfahrung und Voraussage berufen. [Weyl 1953, 226]

This approach is far from the notion of intuition as sense perception, but it is close to the idea of intuition as symbolic construction. Again, the basic idea would be that the procedures used in the construction of a symbolic system are those which provide its epistemological justification.

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